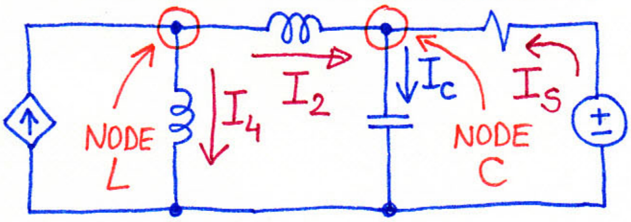
Q1

4 NODES : 1 REFERENCE NODE $V_{REF} = 0V$
1 TRIVIAL NODE $V_S = 10V$ → ALTERNATIVELLY, TRANSFORM INTO CURRENT SOURCE TO ELIMINATE
1 NODE, NO NEED TO CALC V_L
1 NODE, CALC $V_C = ?$
} 2 EQUATIONS W/ 2 UNKNOWNS SOLVE FOR V_C ONLY

1 ANALYSIS

ASSUMED CURRENT DIRECTIONS:

1 S/D



KCL FOR NODE L (2)

$$3I_C = I_4 + I_2$$

KCL FOR NODE C (2)

$$I_2 + I_S = I_C$$

CURRENTS:

$$3I_C = 3\frac{V_C}{Z_C} = \frac{3}{-j1}V_C = j3V_C \quad (1)$$

$$I_4 = \frac{V_L}{Z_{L4}} = \frac{1}{j4}V_L = -j0.25V_L \quad (1)$$

$$I_2 = \frac{V_L - V_C}{Z_{L2}} = \frac{1}{j2}(V_L - V_C) = -j0.5(V_L - V_C) \quad (1)$$

$$I_S = \frac{V_S - V_C}{R} = \frac{1}{8}(10 - V_C) = 0.125(10 - V_C) \quad (1)$$

$$I_C = \frac{V_C}{Z_C} = \frac{1}{-j1}V_C = jV_C \quad (0.67)$$

PLUG IN CURRENTS IN KCL EQUATIONS

$$j3V_C = -j0.25V_L - j0.5(V_L - V_C) = -j0.25V_L - j0.5V_L + j0.5V_C$$

2 EQ.#1

$$\boxed{j2.5V_C + j0.75V_L = 0} \quad \text{EQ. \#1} \Longrightarrow V_L = \frac{-j2.5V_C}{j0.75} = -\frac{10}{3}V_C$$

$$-j0.5(V_L - V_C) + 0.125(10 - V_C) = jV_C$$

$$-j0.5V_L + j0.5V_C + 1.25 - 0.125V_C - jV_C = 0$$

$$(-0.125 - j0.5)V_C - j0.5V_L = -1.25 \quad / \cdot (-1)$$

PLUG IN V_L IN EQ.#2

2 EQ.#2

$$\boxed{(0.125 + j0.5)V_C + j0.5V_L = 1.25} \quad \text{EQ. \#2}$$

$$(0.125 + j0.5)V_C - j0.5 \cdot \frac{10}{3}V_C = 1.25$$

$$0.125V_C + j0.5V_C - j\frac{5}{3}V_C = 1.25$$

$$0.125V_C - j\frac{3.5}{3}V_C = 1.25$$

FINAL SOLUTION (2)

$$\boxed{V_C =} \frac{1.25}{0.125 - j\frac{3.5}{3}} = \frac{1.25 \cdot 0.125 + j\frac{1.25 \cdot 3.5}{3}}{0.125^2 + \left(\frac{3.5}{3}\right)^2} = \boxed{0.1135 + j1.06\ V}$$

Q2

$V_{S1} = 6\sin(3t) = 6\cos(3t - 90°) \Rightarrow V_{S1} = 6\angle{-90°}\ V$

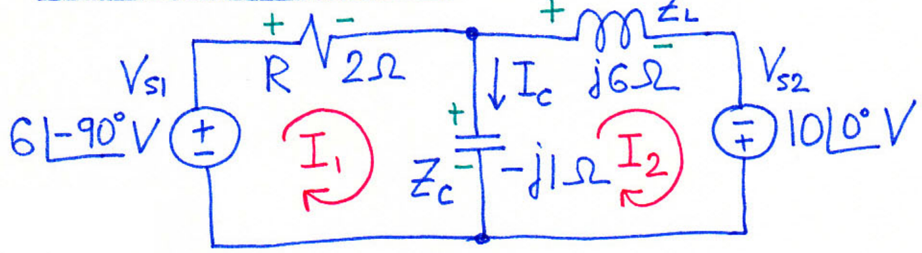
$V_{S2} = 10\cos(3t) \Rightarrow V_{S2} = 10\ V$

TRANSFORM SIN → COS 2

$\omega = 3\ \frac{rad}{s}$ ← READ OUT ω 1

$\Rightarrow R = 2\Omega\ ;\ X_L = \omega L = 3 \cdot 2 = 6\Omega$ 1 $;\ X_c = -\frac{1}{\omega C} = -\frac{1}{3 \cdot 0.333} = -1\Omega$ 1

THUS THE CIRCUIT IS:



DESIGNATE VOLTAGES AND MESH CURRENTS 2

I_c IS REAL (ACTUAL) CURRENT: $I_c = I_1 - I_2$ 1

WRITE
- KVL LOOP 1: $-V_{S1} + RI_1 + Z_c(I_1 - I_2) = 0$ 2 EQ. #1
- KVL LOOP 2: $-V_{S2} - Z_c(I_1 - I_2) + Z_L I_2 = 0$ 2 EQ. #2

SORT
- 1: $6\angle{-90°} = 2I_1 - jI_1 + jI_2 = (2 - j1)I_1 + jI_2$
- 2: $10\angle{0°} = jI_1 - jI_2 + j6I_2 = j1 \cdot I_1 + j5I_2$

$I_2 = \frac{10 - jI_1}{j5} = -\frac{I_1}{5} - j2$ 0.67

SOLVE

1: $6\angle{-90°} = -j6 = (2 - j1)I_1 + j\left(-\frac{I_1}{5} - j2\right)$

$-j6 = (2 - j1)I_1 - j\frac{1}{5}I_1 + 2$

$-2 - j6 = (2 - j1.2)I_1$

$\hookrightarrow I_1 = \frac{-2 - j6}{2 - j1.2} \cdot \frac{2 + j1.2}{2 + j1.2} = \frac{-4 + 7.2 - j2.4 - j12}{2^2 + 1.2^2} = \frac{3.2 - j14.4}{5.44}$

$I_1 = 0.5882 - j2.647\ A$ 1

2: $I_2 = -\frac{I_1}{5} - j2 = -0.2(0.5882 - j2.647) - j2$

$= -0.11764 + j0.5294 - j2$

$= -0.11764 - j1.4706\ A$ 1

FINALLY:

$I_c = I_1 - I_2 = 0.5882 - j2.647 + 0.11764 + j1.4706$

$$\boxed{I_c = 0.70584 - j1.1764\ A}$$

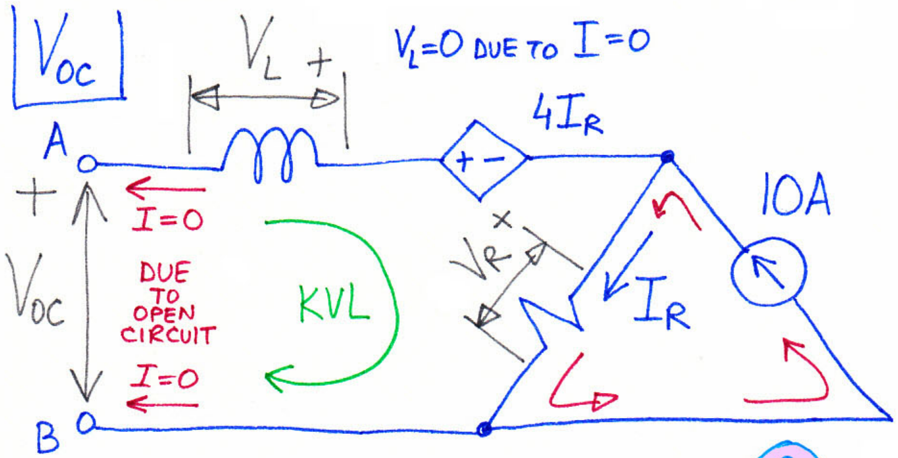
2 FINAL SOLUTION

Q3

CIRCUIT CONTAINS DEPENDENT SOURCE
$\Rightarrow$ MUST FIND BOTH V_{oc} & I_{sc} 1

V_{oc}

$V_L = 0$ DUE TO $I = 0$



$I_R = 10A$ 1
CIRCULATING CURRENT

$V_R = I_R \cdot R$
$= 10A \cdot 2\Omega$
$= 20V$ 1

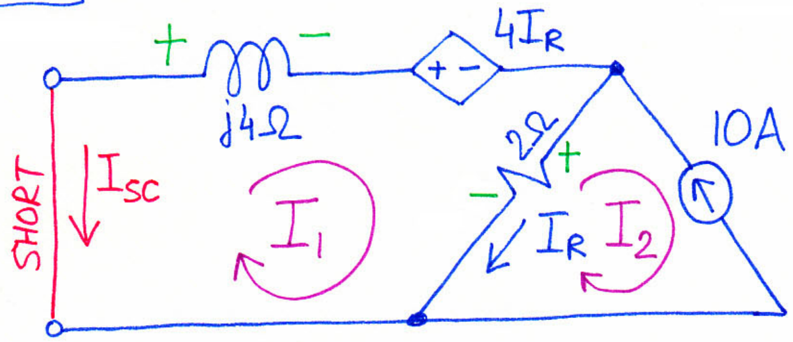
KVL: $-V_{oc} - V_L + 4I_R + V_R = 0$ 2

$V_{oc} = -0 + 4 \cdot 10 + 20 \Rightarrow \boxed{V_{oc} = 60V}$

$V_{TH} = V_{oc} = 60V$ 2

I_{sc}

REDRAW CIRCUIT:



$I_2 = -10A$ ✓ 1

$I_R = I_1 - I_2$
$= I_1 - (-10A)$
$= I_1 + 10A$ 1

CANNOT USE CURRENT DIVIDER DUE TO (DEPENDENT) SOURCE. 1
USE MESH CURRENTS:

KVL LOOP 1 : $+Z_L I_1 + 4I_R + RI_R = 0$ 2 KVL EQUATION

$j4I_1 + 4(I_1 + 10) + 2(I_1 + 10) = 0$

$j4I_1 + 6I_1 + 60 = 0$

$(6 + j4)I_1 = -60$

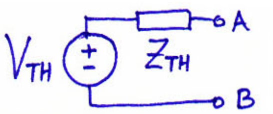
THUS $I_1 = \frac{-60}{6+j4} \cdot \frac{6-j4}{6-j4} = \frac{-360+j240}{36+16} = -\frac{90}{13} + j\frac{60}{13}A$

$$\boxed{I_{sc} = -I_1 = \frac{90}{13} - j\frac{60}{13}A}$$ 2

Z_{TH}

$Z_{TH} = \frac{V_{oc}}{I_{sc}} = \frac{60}{\frac{90}{13} - j\frac{60}{13}} = \frac{6 \cdot 13}{9 - j6} \cdot \frac{9+j6}{9+j6} = \frac{702+j468}{9^2+6^2} = 6 + j4\Omega$

$Z_{TH} = 6 + j4\Omega$ 2



0.67

Q4

PERIOD $T = 6\,s$ (2)

$$v(t) = \begin{cases} -4V & 0 \le t < 3 \\ 2V & 3 \le t < 6 \end{cases}$$ (2) (2)

CONSTANT VALUE IN EACH SEGMENT

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^6 v^2_{(t)}\,dt} \;(2)\; = \sqrt{\frac{1}{6}\left(\int_0^3 (-4)^2 dt + \int_3^6 2^2 dt\right)} \;(2)$$

$$= \sqrt{\frac{1}{6}\left(16\cdot t\Big|_0^3 + 4\cdot t\Big|_3^6\right)} \;(2)\; = \sqrt{\frac{1}{6}\left(16\cdot 3 + 4(6-3)\right)} \;(0.67)$$

FINAL (4)

$$= \sqrt{\frac{1}{6}(48+12)} = \sqrt{\frac{60}{6}} = \sqrt{10} \Rightarrow \boxed{V_{RMS} = 3.16\,V}$$

Q5

SERIES RLC RESONANT CIRCUIT

a) RESONANT FREQUENCY: $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{4\cdot 10^{-3}\cdot 4\cdot 10^{-9}}} = \frac{1}{8\pi}\cdot 10^6\,Hz$

THUS $\boxed{f_0 = 39.8\,kHz}$ (1.67)

QUALITY FACTOR: $Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{4}\sqrt{\frac{4\cdot 10^{-3}}{4\cdot 10^{-9}}} = \frac{10^3}{4} \Rightarrow \boxed{Q = 250}$ (1.5)

BANDWIDTH: $B = \frac{f_0}{Q} = \frac{39.8\,KHz}{250} = 159.155\,Hz \Rightarrow \boxed{B = 159.2\,Hz}$ (1.5)

b) HALF-POWER FREQUENCIES

$$f_{LOW} = f_0\left[-\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right] = f_0\left[-\frac{1}{500} + \sqrt{\left(\frac{1}{500}\right)^2 + 1}\right] = f_0\cdot 0.998$$

f_0 ← 39.8 kHz

$\Rightarrow \boxed{f_{LOW} = 39.7204\,kHz}$ (2)

$B = f_{HIGH} - f_{LOW} \Rightarrow f_{HIGH} = f_{LOW} + B = 39{,}720.4 + 159.2$

$\Rightarrow \boxed{f_{HIGH} = 39.8796\,kHz}$ (1)

c) CURRENT MAGNITUTE @ RESONANCE

$Z_L + Z_C = 0 \Rightarrow I = \frac{V}{R} = \frac{2V_{RMS}}{4\Omega} \Rightarrow \boxed{I = 500\,mA_{RMS}}$ (3)

d) $\overline{V}_C$ & $\overline{V}_L$

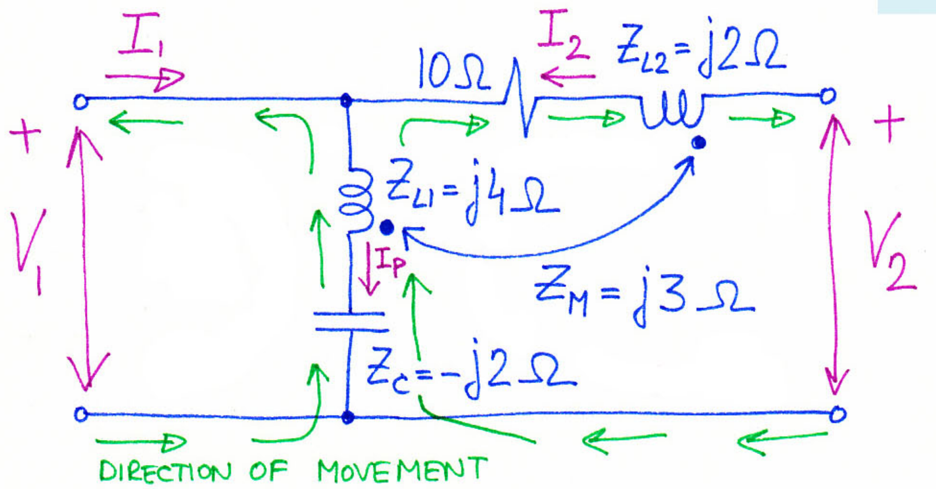
IF $\overline{V}_S$ IS IN REFERENCE $\Rightarrow$

$$\boxed{\begin{aligned} \overline{V}_C &= Q\overline{V}_S \angle -90^\circ = 500\angle -90^\circ\,V_{RMS} \\ \overline{V}_L &= Q\overline{V}_S \angle 90^\circ = 500\angle 90^\circ\,V_{RMS} \end{aligned}}$$ (1.5) (1.5)

e) 10 TIMES LOWER Q

$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \sim \frac{1}{\sqrt{C}}$ CAPACITANCE MUST INCREASE 100 TIMES $\boxed{C_{NEW} = 0.4\,\mu F}$ (3)

Q6



I_P FLOWS OUT OF DOT OF L1

I_2 FLOWS INTO DOT OF L2

$I_P = I_1 + I_2$

HINT ON WRITING EQUATIONS TUTORIAL #10, PROBLEM 6

ASSUME STANDARD 2-PORT NETWORK DIRECTIONS FOR I_1 AND I_2.

Z PARAMETER EQUATIONS:

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

2

WRITE EQUATIONS FOR V_1 AND V_2 ACCORDING TO RULES FOR MAGNETICALLY COUPLED INDUCTORS:

UPSTREAM

LOCAL & REMOTE CURRENTS DO NOT MATCH (OPPOSITE SIGN FROM SELF TERM)

$$V_1 = \underbrace{+Z_C I_P}_{\text{CAP}} \underbrace{+ Z_{L1} I_P}_{\text{SELF}} \underbrace{- Z_M I_2}_{\text{MUTUAL}}$$

4

$$= (Z_C + Z_{L1}) I_P - Z_M I_2$$
$$= (-j2 + j4)(I_1 + I_2) - j3 I_2$$
$$= j2 I_1 + j2 I_2 - j3 I_2 = \underbrace{(j2)}_{Z_{11}} I_1 + \underbrace{(-j1)}_{Z_{12}} I_2$$

1 1

$$V_2 = \underbrace{+ Z_C I_P}_{\text{CAP}} \underbrace{+ Z_{L1} I_P}_{\substack{\text{SELF} \\ \text{L1}}} \underbrace{- Z_M I_2}_{\substack{\text{MUTUAL} \\ (I_2 \text{ REMOTE})}} \underbrace{+ R I_2}_{\text{R}} \underbrace{+ Z_{L2} I_2}_{\substack{\text{SELF} \\ \text{L2}}} \underbrace{- Z_M I_P}_{\substack{\text{MUTUAL} \\ (I_P \text{ REMOTE})}}$$

6

$$= (Z_C + Z_{L1} - Z_M) I_P + (R - Z_M + Z_{L2}) I_2$$
$$= (-j2 + j4 - j3)(I_1 + I_2) + (10 - j3 + j2) I_2$$
$$= -j1 I_1 - j1 I_2 + (10 - j1) I_2$$
$$= \underbrace{(-j1)}_{Z_{21}} I_1 + \underbrace{(10 - j2)}_{Z_{22}} I_2$$

1 1

THUS Z MATRIX IS:

$$Z = \begin{vmatrix} j2 & -j1 \\ -j1 & 10-j2 \end{vmatrix}$$

0.67

Q7

TRANSFER FUNCTION : (APPLY VOLTAGE DIVIDER RULE)

$$\bar{H} = \frac{\bar{V}_{OUT}}{\bar{V}_{IN}} = \frac{Z_R}{Z_R + \bar{Z}_L} = \frac{R}{R + j\omega L} \cdot \frac{\frac{1}{R}}{\frac{1}{R}} = \frac{1}{1 + j\omega \frac{L}{R}} \quad (2)$$

$$\omega_p = \frac{R}{L}$$

PLUG IN R AND L VALUES :

$$\omega_p = \frac{R}{L} = \frac{10\,\Omega}{0.1\,mH} = 100k\ \frac{rad}{s} \quad (2)$$

STANDARD FORM :

$$\boxed{H(j\omega)} = \frac{1}{1 + j\frac{\omega}{\omega_p}} = \boxed{\frac{1}{1 + j\frac{\omega}{100k}}} \quad (2.67)$$

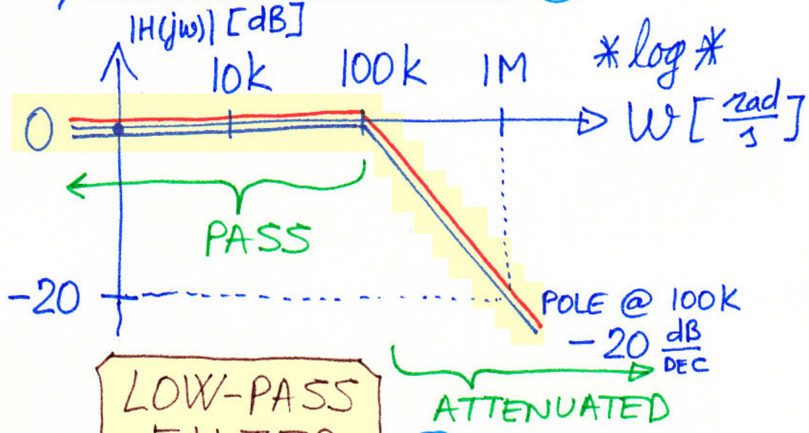
$K_0 = 1$

"EMPTY" NUMERATOR

1^{ST} ORDER TERM IN DENOMINATOR

CORNER FREQUENCY ω_p

a) MAGNITUTE PLOT (4)



ONLY ONE COMPONENT. FINAL PLOT IS THE SAME AS 1^{ST} ORDER TERM PLOT.

$\boxed{\text{LOW-PASS FILTER}}$ (2)

ANSWER TO b)

PHASE PLOT (4)