Question 6.

For the 2-port network with magnetically coupled coils shown in *Figure 6*, calculate z parameters.

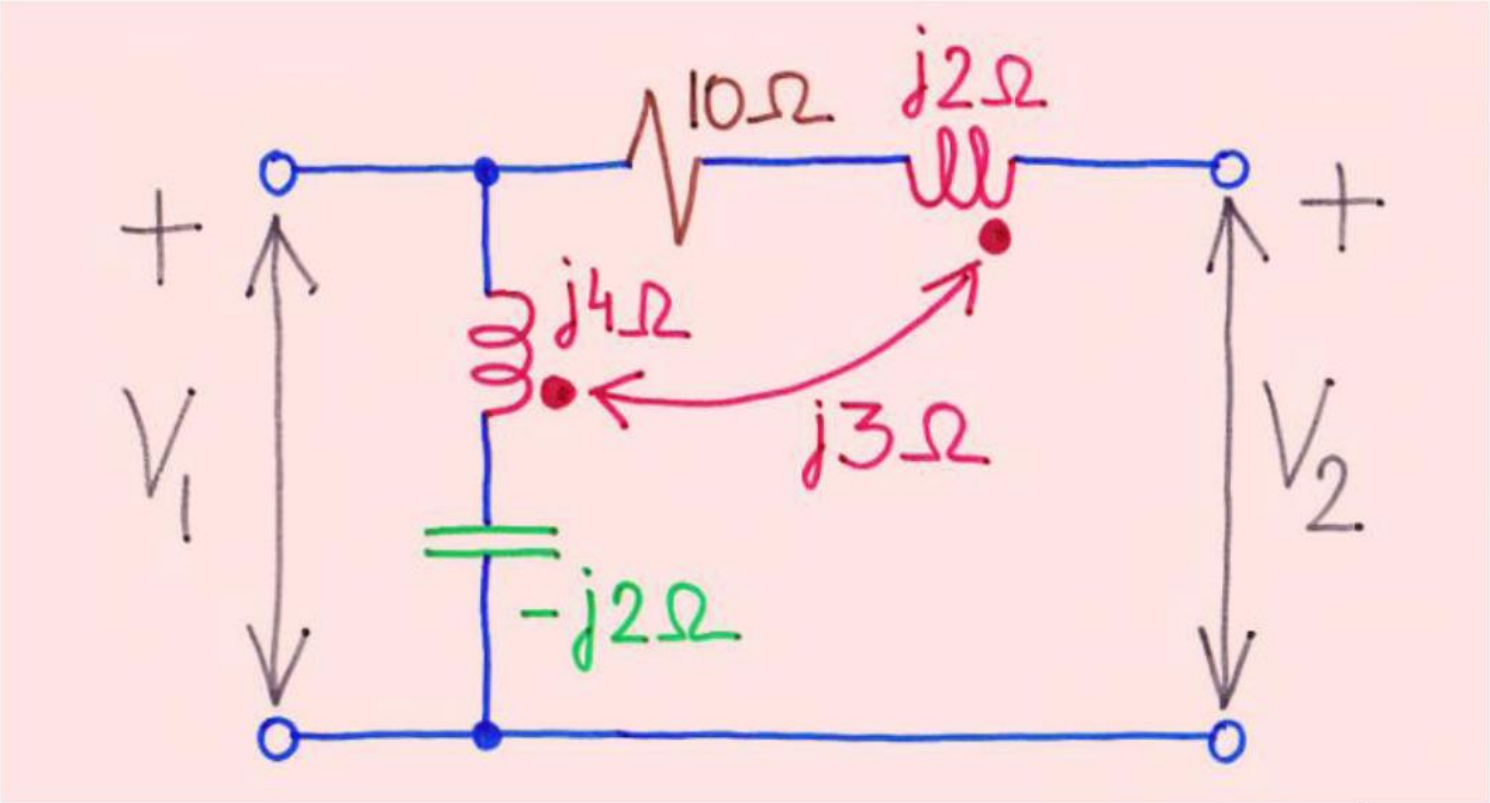
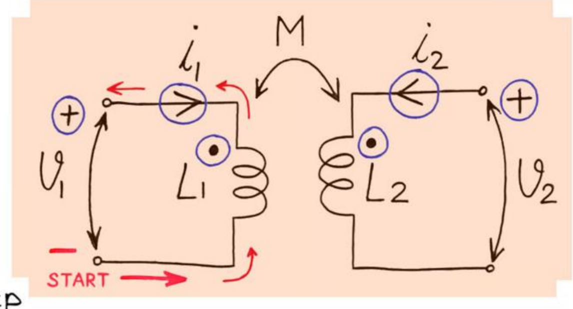


Figure 6

MAGNETIC COUPLING



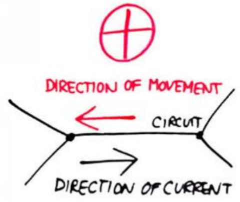
"STANDARD" OR "BASE CASE" :

PASSIVE SIGN CONVENTION (BETWEEN CURRENTS AND VOLTAGES) (ON BOTH SIDES)

- THE VOLTAGE "+" IS AT THE DOT (FOR BOTH v_1 & v_2)
- THE CURRENT FLOWS INTO THE DOT (FOR BOTH i_1 & i_2)

STEP

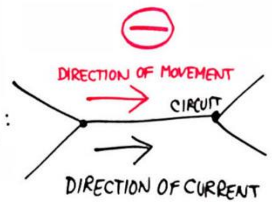
1. WE WANT TO WRITE EQUATION FOR VOLTAGE (E.G. $v_1 = \dots$)
2. START AT "−" AND MOVE THRU CIRCUIT TOWARDS "+"
3. $\boxed{\text{SELF COMPONENT}}$ VOLTAGE "SELF" TERM — LOCAL CURRENT DIRECTION MATTERS. DOTS CAN BE DISREGARDED HERE.
   - UPSTREAM :



UPSTREAM = POSITIVE

$\oplus\, L\dfrac{di}{dt}$ ← LOCAL

   - DOWNSTREAM :



$\ominus\, L\dfrac{di}{dt}$ ← LOCAL

DOWNSTREAM = NEGATIVE

4. $\boxed{\text{MUTUAL EFFECT}}$ VOLTAGE "MUTUAL" TERM — LOCAL & REMOTE CURRENT DIRECTIONS MATTER. DOTS DO MATTER.
   - BOTH CURRENTS FLOW INTO (OUT OF) DOTS

     $\boxed{\text{THE SAME}}$ SIGN AS SELF COMPONENT

     $* \, M\dfrac{di}{dt}$ ← REMOTE

     IF CURRENT DIRECTIONS $\boxed{\text{MATCH}}$ THE DOTS, THEN "MUTUAL" SIGN $\boxed{\text{MATCHES}}$ THE "SELF" SIGN

   - ONE CURRENT INTO DOT, THE OTHER OUT OF

     $\boxed{\text{THE OPPOSITE}}$ SIGN FROM SELF COMPONENT

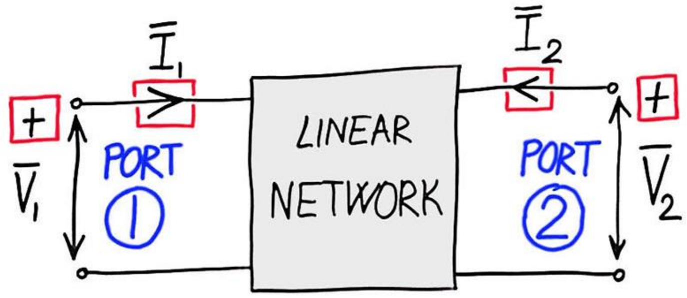
     $* \, M\dfrac{di}{dt}$ ← REMOTE

     IF CURRENT DIRECTIONS $\boxed{\text{DO NOT MATCH}}$ THE DOTS, THEN "MUTUAL" SIGN IS $\boxed{\text{OPPOSITE}}$ FROM THE "SELF" SIGN

time: $L\dfrac{di(t)}{dt}$

PHASOR: $\bar{Z}_L \cdot \bar{I}$ $\quad \bar{Z}_L = j\omega L$ $\quad \bar{Z}_M = j\omega M$

2-PORT NETWORKS



Z:

$$\bar{V}_1 = Z_{11}\bar{I}_1 + Z_{12}\bar{I}_2$$
$$\bar{V}_2 = Z_{21}\bar{I}_1 + Z_{22}\bar{I}_2$$

$$Z_{11} = \left.\frac{\bar{V}_1}{\bar{I}_1}\right|_{\bar{I}_2=0} \qquad Z_{12} = \left.\frac{\bar{V}_1}{\bar{I}_2}\right|_{\bar{I}_1=0}$$

$$Z_{21} = \left.\frac{\bar{V}_2}{\bar{I}_1}\right|_{\bar{I}_2=0} \qquad Z_{22} = \left.\frac{\bar{V}_2}{\bar{I}_2}\right|_{\bar{I}_1=0}$$

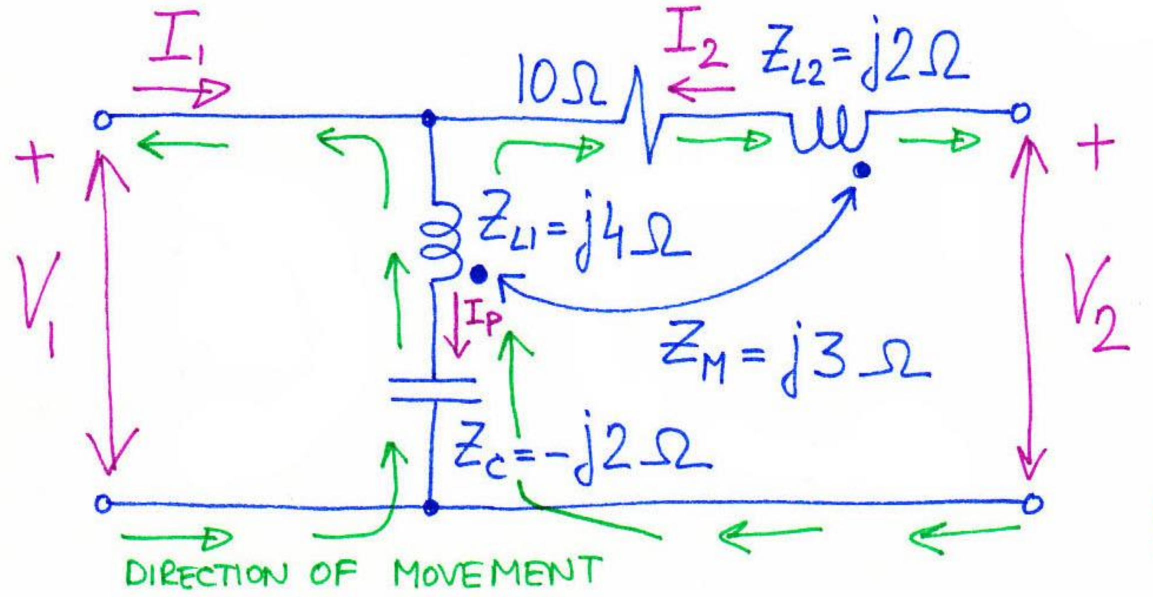
Y:

$$\bar{I}_1 = y_{11}\bar{V}_1 + y_{12}\bar{V}_2$$
$$\bar{I}_2 = y_{21}\bar{V}_1 + y_{22}\bar{V}_2$$

$$y_{11} = \left.\frac{\bar{I}_1}{\bar{V}_1}\right|_{\bar{V}_2=0} \qquad y_{12} = \left.\frac{\bar{I}_1}{\bar{V}_2}\right|_{\bar{V}_1=0}$$

$$y_{21} = \left.\frac{\bar{I}_2}{\bar{V}_1}\right|_{\bar{V}_2=0} \qquad y_{22} = \left.\frac{\bar{I}_2}{\bar{V}_2}\right|_{\bar{V}_1=0}$$

Q6



I_P FLOWS OUT OF DOT OF L1

I_2 FLOWS INTO DOT OF L2

$$I_P = I_1 + I_2$$

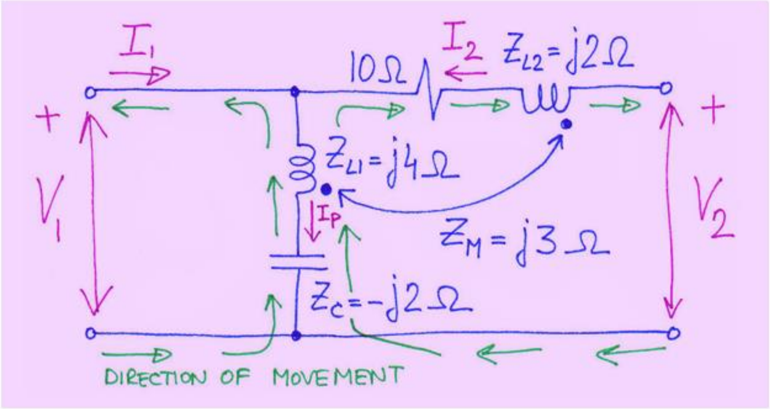
HINT ON WRITING EQUATIONS TUTORIAL #10, PROBLEM 6

ASSUME STANDARD 2-PORT NETWORK DIRECTIONS FOR I_1 AND I_2.

Z PARAMETER EQUATIONS:

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

Magnetically Coupled Inductors – Solution (page 2)



WRITE EQUATION FOR V_1 ACCORDING TO RULES FOR MAGNETICALLY COUPLED INDUCTORS:

UPSTREAM

LOCAL & REMOTE CURRENTS DO NOT MATCH (OPPOSITE SIGN FROM SELF TERM)

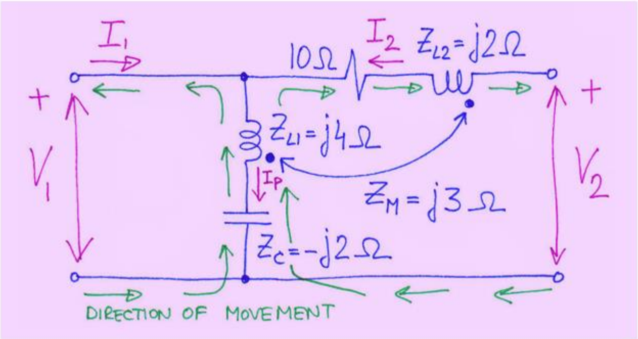
$$V_1 = \underbrace{+Z_C I_P}_{\text{CAP}} \underbrace{+Z_{L1} I_P}_{\text{SELF}} \underbrace{-Z_M I_2}_{\text{MUTUAL}}$$

$$= (Z_C + Z_{L1}) I_P - Z_M I_2$$

$$= (-j2 + j4)(I_1 + I_2) - j3 I_2$$

$$= j2 I_1 + j2 I_2 - j3 I_2 = \overbrace{(j2)}^{Z_{11}} I_1 + \overbrace{(-j1)}^{Z_{12}} I_2$$

Magnetically Coupled Inductors – Solution (page 3)



WRITE EQUATION FOR V_2 ACCORDING TO RULES FOR MAGNETICALLY COUPLED INDUCTORS:

$$V_2 = \underbrace{+Z_C I_P}_{\text{CAP}} \underbrace{+Z_{L1} I_P}_{\substack{\text{SELF}\\ \text{L1}}} \underbrace{-Z_M I_2}_{\substack{\text{MUTUAL}\\ (I_2 \text{ REMOTE})}} \underbrace{+R I_2}_{\text{R}} \underbrace{+Z_{L2} I_2}_{\substack{\text{SELF}\\ \text{L2}}} \underbrace{-Z_M I_P}_{\substack{\text{MUTUAL}\\ (I_P \text{ REMOTE})}}$$

$$= (Z_C + Z_{L1} - Z_M) I_P + (R - Z_M + Z_{L2}) I_2$$

$$= (-j2 + j4 - j3)(I_1 + I_2) + (10 - j3 + j2) I_2$$

$$= -j1 I_1 - j1 I_2 + (10 - j1) I_2$$

$$= \underbrace{(-j1)}_{Z_{21}} I_1 + \underbrace{(10 - j2)}_{Z_{22}} I_2$$

THUS Z MATRIX IS:

$$Z = \begin{vmatrix} j2 & -j1 \\ -j1 & 10-j2 \end{vmatrix}$$